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Atmospheric Neutrino Event Rates: The Expectations

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Atmospheric neutrino event rates: the expectations

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Naturally occurring energetic neutrinos produced by interactions of cosmic rays in the atmosphere produce a signal in deep underground detectors which is a convolution of neutrino flux, neutrino cross section and detection efficiency. The predicted ratio of events induced by ν_e as compared to ν_μ is relatively robust (because many sources of uncertainty cancel in the ratio), but it differs significantly from what is observed.

Interpretations that involve new physics (e.g. neutrino oscillations or nucleon decay) have been proposed. One interpretation in terms of neutrino oscillations would imply a low value of neutrino-induced upward muons. Although there is no strong evidence for such a deficit, uncertainties in calculating the expected absolute rate prevent one from eliminating this interpretation at present. Precise measurement of muon fluxes at high altitude, as well as calibration of detectors with neutrino beams from accelerators, will help clarify the situation.

1. Introduction

Hess discovered the cosmic radiation of extraterrestrial origin (Hess 1912) by measuring the effects of secondary interactions of cosmic rays in the atmosphere. Over the next 35 years the understanding of the various secondary components developed in close connection with related discoveries in particle physics, as documented by Hillas (1972). Figure 1 shows the main components of the cosmic radiation in the atmosphere. Because they interact only through the weak interaction, neutrinos were the last secondary component to be measured, even though they are as abundant as muons deep in the atmosphere.

The idea of detecting neutrinos by looking for neutrino-induced upward or horizontal muons was suggested by Markov (1960). The process is

$$\nu_\mu + \mathrm{N} \rightarrow \mu + \text{anything}, \tag{1}$$

where N is a nucleon in the material surrounding the detector. The muon range increases with energy. This extends the effective target volume and makes it possible to see neutrino-induced muons with detectors of moderate size. At about the same time Greisen (1960) described a neutrino detector very much like the current large water Cherenkov detectors, and he estimated the event rate to be expected for interactions of atmospheric neutrinos inside a 3 kt sensitive volume. Two groups (Achar *et al.* 1965; Reines *et al.* 1965) reported the first observations of atmospheric neutrinos with the detection of horizontal muons in detectors so deep that the muons could not have been produced in the atmosphere. These results were among the subjects of an earlier Royal Society Discussion Meeting in 1966 (Menon *et al.* 1967; Reines 1967).

Atmospheric neutrinos are of current interest, despite their long history and apparently mundane origin, because of the anomalous flavour ratio observed for

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Figure 1

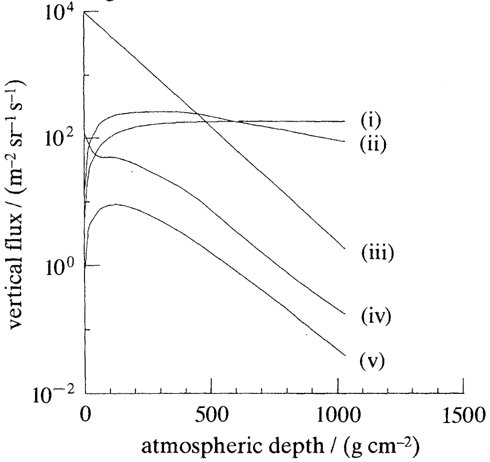

Figure 2

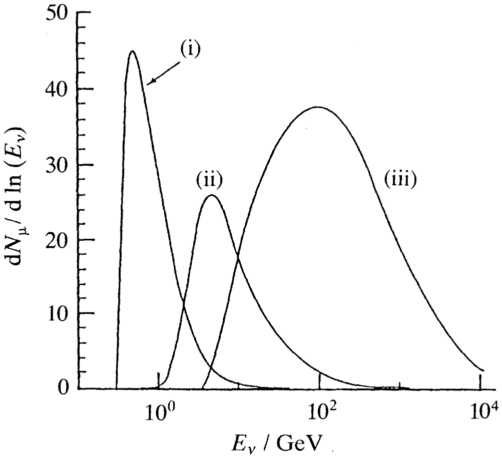

Figure 1. Vertical fluxes of cosmic rays in the atmosphere. The curves show the integral intensities of particles with energies greater than 1 GeV: (i) neutrinos (muon), (ii) muons, (iii) nucleons, (iv) electrons, (v) charged pions.

Figure 2. Distribution of energies of muon neutrinos (plus anti-neutrinos) that give rise to contained events, upward muons with $E_\mu < 4$. GeV and upward muons with $E_\mu > 4$ GeV. The average over solid angle is taken. (i) Contained interactions ($\times\frac{1}{10}$), (ii) stopping muons, (iii) throughgoing muons.

neutrino interactions in the large proton decay detectors (Hirata *et al.* 1992; Becker-Szendy *et al.* 1992*a*). The experimental situation is the subject of Beier & Frank's paper in this volume. The essential point is that, because of their large volume, these detectors can measure interactions of neutrinos inside the detector. They need not depend on the large external target mass provided by the long range of energetic muons produced in charged current interactions of muon-type neutrinos. They can therefore study both ν_e and ν_μ interactions. The anomaly is that the observed ratio of events produced by electron neutrinos to those from muon neutrinos is significantly larger than expected.

For both kinds of events the signal can be written as the convolution of neutrino flux, neutrino cross section and detection efficiency,

$$\text{signal} = \phi_\nu \otimes \sigma_\nu \otimes \epsilon.$$

In the case of events of external origin, the detection efficiency ϵ contains the range of the produced muons as well as the projected effective area of the detector. Figure 2 shows the distribution of energies of the neutrinos that produce three classes of events: (1) interactions of $\nu_\mu + \bar{\nu}_\mu$ inside the detector, (2) neutrino-induced upward muons (zenith angle greater than 90°) that enter the detector and stop (here $E_\mu <$ 4 GeV) and (3) upward muons with $E > 4$ GeV.

For most of the interactions inside the detector the neutrino energy is less than 1 GeV. In the large water Cherenkov detectors nearly all the neutrino interactions occur on oxygen nuclei rather than free nucleons. The Fermi gas model (FGM) has been used in obtaining the configurations of the neutrino events for studies of detector response and interpretation of the data. The adequacy of this simple model in this context has been questioned from time to time. There are two indications that use of the FGM is not the source of the anomalous flavour ratio. Merenyi *et al.* (1992)

show that data taken on deuterium and neon targets with $E_\nu > 400$ MeV are consistent with what is expected in the FGM for atmospheric neutrinos (Gaisser & O'Connell 1986). On the theoretical side, Engel *et al.* (1993) have investigated several nuclear effects beyond the FGM. They find a negligible effect on the predicted ratio of ν_e/ν_μ interactions.

This paper emphasizes the calculation of the flux of atmospheric neutrinos, both in the low energy region relevant to internal events and in the higher energy region important for external events. The subject of neutrino-induced upward muons and their implications for interpretation of the contained event anomaly is also discussed.

2. Neutrino flux

In the early 1960s there were several increasingly detailed calculations of the intensity of atmospheric neutrinos for $E_\nu \geqslant 1$ GeV (Markov & Zheleznykh 1961; Zatsepin & Kuzmin 1962; Cowsik *et al.* 1966; Osborne *et al.* 1965). Tam & Young (1970) extended the calculations down to 200 MeV. Some of these calculations inferred the production spectrum of parent pions from the observed muon flux, then used the pion spectrum to derive the neutrino flux. More recent calculations start from the primary cosmic ray spectrum and calculate both muons and neutrinos. In either case, the comparison with measured fluxes of atmospheric muons provides an important check on the calculations.

(*a*) *Low energy region*

Here I will compare four detailed calculations of the low energy neutrino flux with the aim of estimating the uncertainty in the expected neutrino flux. The calculation of Barr *et al.* (1989, hereafter called BGS) began with the work of Gaisser *et al.* (1983). It is a one-dimensional Monte Carlo calculation that starts from the primary spectrum, uses a fit of inclusive cross sections to accelerator data (Eichten *et al.* 1972) and assumes all secondaries propagate along the direction of the primary. The calculation is described in detail by Gaisser *et al.* (1988).

Lee & Koh (1990, LK) use an interaction model related to that of Gaisser *et al.* (1983, 1988) extended to include transverse momentum of the secondaries. They then make a three-dimensional calculation of the neutrino flux. In a preliminary version of the calculation (Lee & Bludman 1988) they state that the result is the same as the one-dimensional calculation for $E_\nu > 200$ MeV. The common origin of the hadronic production model does not guarantee that the results will be the same as BGS above 200 MeV because LK independently treat the geomagnetic cut-offs.

The other calculations are completely independent of the first two and of each other. The work of Honda *et al.* (1990, HKHM) is a Monte Carlo calculation that includes a detailed treatment of the effect of the geomagnetic field. Cut-offs are calculated for each detector location by propagating antiprotons outward from Earth through a detailed map of the geomagnetic field along a set of initial directions centred on each detector. The calculation of Bugaev & Naumov (1989, BN) is a numerical integration of the cascade equations. All four calculations include the effect of muon polarization on the neutrinos from muon decay, following the remark of Volkova (1989) who emphasized its importance in this context. (The fluxes shown by Bugaev & Naumov (1989) do not include muon polarization, but they have since been corrected for this effect (V. A. Naumov, private communication).)

Table 1. *Calculated neutrino fluxes at Kamiokande normalized to BGS*

	$\nu_\mu+\bar{\nu}_\mu$			$\nu_e+\bar{\nu}_e$		
E_ν/GeV	HKHM	LK	BN	HKHM	LK	BN
0.4–1	0.90	0.79	0.63	0.87	0.79	0.62
1–2	0.95	0.80	0.79	0.91	0.81	0.74
2–3	1.04	0.81	0.95	0.97	0.83	0.87

Table 2. *Neutrino ratios* $0.4 < E_\nu < 1$ GeV

ref.	R_μ	R_e	$R_{e/\mu}$
BGS	0.99	0.89	0.49
HKHM	0.99	0.84	0.48
LK	1.00	0.99	0.48
BN	0.98	0.76	0.50

The tables 1 and 2 illustrate the differences and similarities among the flux calculations. Table 1 gives the ratio of the neutrino fluxes in three energy intervals compared to the BGS calculation. The biggest discrepancy is between the Bugaev & Naumov calculation and that of BGS, which differ by almost a factor of two in the lowest energy bin. As energy increases, however, the two calculations give similar results. Bugaev *et al.* (1986) show a compilation of measurements (Bogomolov *et al.* 1979) of the high altitude (11–16 km) intensity of muons with kinetic energy in the range 100 MeV to 1 GeV. Stanev (1993) has compared the muon fluxes from the BGS calculation to the same data. As with the neutrino fluxes, the BGS muon fluxes are significantly higher than those of Bugaev *et al.* below 1 GeV, and they approach each other as energy increases. The higher muon intensities appear to agree somewhat better with the data, though the experimental uncertainties are large.

The second table is similar to a figure of Honda (1993) in which he compared ratios of neutrinos from the four flux calculations. Here I have tabulated $R_{e/\mu} = (\nu_e+\frac{1}{3}\bar{\nu}_e)/(\nu_\mu+\frac{1}{3}\bar{\nu}_\mu)$ because the cross section for quasi-elastic interactions of antineutrinos is roughly one-third that of the neutrinos in the low energy range relevant for single-ring contained events. The anti-neutrino/neutrino ratios in table 2 are $R_\mu = \bar{\nu}_\mu/\nu_\mu$ and $R_e = \bar{\nu}_e/\nu_e$. The results of the comparisons made here can be summarized as follows.

1. In the energy range between 0.1 and 1 GeV $R_\mu \approx 1$, as expected from the kinematics of pion and muon decay, which causes the secondary ν_μ from pion decay to have roughly the same energy on average as the tertiary $\bar{\nu}_\mu$ from muon decay.
2. $R_e < 1$ as expected from the predominance of positive charge among the fast secondaries (as in the μ^+/μ^- ratio). The value $R_e \approx 1$ obtained by LK is anomalous.
3. $R_{e/\mu} \approx 0.49 \pm 0.01$.

Although the fluxes at low energy differ by as much as 50%, the ν_e/ν_μ ratio is the same within 5% in the four calculations. This is a consequence of the fact that the main uncertainties in the calculation cancel to first order when the ratio is taken. Second-order effects remain.

1. The primary energy spectrum. A softer spectrum will give fewer high energy neutrinos, which can affect the measured ratio in two ways, first because the

threshold for detection of muons is higher than for electrons and second because the relative weights of ν_μ and ν_e depend on the spectral slope.

2. The primary composition. The ratio of neutrons to protons affects the ratio $\nu_e/\bar{\nu}_e \approx \mu^+/\mu^-$ because of the tendency of protons to produce more fast π^+ than π^- and vice versa for neutrons.

3. The treatment of the geomagnetic cut-offs. Because nuclei of the same energy per nucleon have twice the magnetic rigidity of free protons, increasing the cut-off rigidity will enhance the contribution of neutrons to the neutrino flux.

Various comparisons among the different calculations have been made in an attempt to identify specific sources of the differences in the expected neutrino fluxes. This effort is in progress at present and was discussed most recently at a workshop at Louisiana State University. The structure of the BGS calculation allows one to test the effect of changing various elements of the calculation one by one. The last step of the BGS calculation is to combine the neutrino yields with the primary spectrum weighted with the geomagnetic cut-offs (separately for free protons and bound nucleons). For example, when we substitute the HKHM cut-offs at Kamioka for those used by BGS, the neutrino fluxes between 0.4 and 1 GeV decrease by about 10%. Use of the HKHM primary spectrum instead of the BGS primary spectrum has an opposite effect of about the same magnitude. Replacing the BGS primary spectrum with that of BN (including an underestimate by BN of the effective number of interacting nucleons from a primary nucleus) decreases the neutrino flux by only about 5% below 1 GeV and increases it by a similar amount above 2 GeV. This is not sufficient to account for the strong suppression of the neutrino flux at low energy in the BN calculation. We believe that the differences among the models arise primarily from differences in the representation of pion production in interactions of nucleons in the atmosphere.

Of primary interest for the contained event anomaly is the ratio $R_{e/\mu}$, which evidently remains stable under changes in primary spectrum, cut-offs and interaction model. The factor in the calculation that most directly affects this ratio is the treatment of muon energy loss and decay, since electron neutrinos in the energy range above *ca.* 100 MeV up to *ca.* 2 GeV come almost entirely from decay in flight of muons. The physics here is completely well-known and unambiguous, but it requires care to implement it correctly in a Monte Carlo program or numerical calculation. In this regard, the fact that several independent calculations give very similar results for $R_{e/\mu}$ is an important check. For the same reason, precision measurements of the intensity and energy spectrum of *ca.* GeV muons as a function of atmospheric depth are important.

(*b*) *High energy region*

Three groups have measured the flux of neutrino-induced upward muons with large detectors (Baksan: Boliev *et al.* 1991, 1993; Kamiokande: Mori *et al.* 1991; and IMB: Becker-Szendy *et al.* 1992*b*). Results will be forthcoming soon from the full MACRO detector (Ronga 1993). Three calculations of the flux of ν_μ have been used in interpreting these experiments, Volkova (1980), Mitsui (Mitsui *et al.* 1986) and Butkevich (Butkevich *et al.* 1988). All of these calculations cover the energy range $E_\nu > 1$ GeV. For comparison I will also discuss an extension of the BGS calculation to high energy (Agrawal *et al.* 1993). A principal motivation of this new calculation is to have a single atmospheric neutrino flux for both low and high energy regions.

A curious feature of the high energy spectrum of muon neutrinos is the increasing

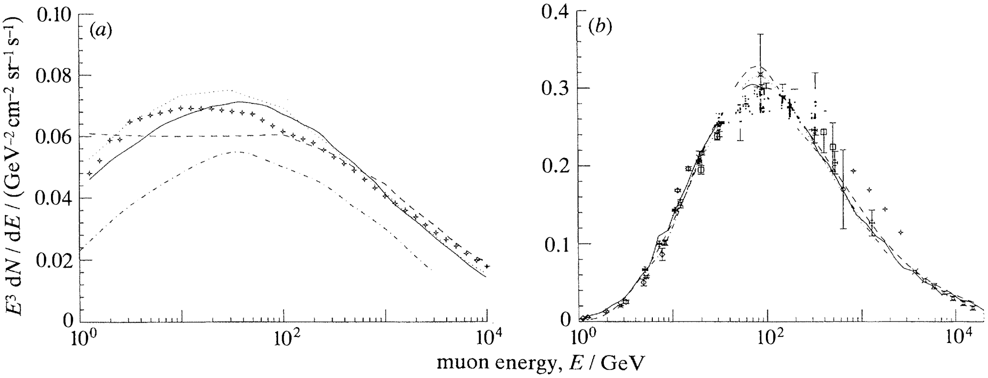

Figure 3. (*a*) Five neutrino flux calculations averaged over solid angle. Solid, Agrawal *et al.* (1993); dotted, Butkevich *et al.* (1988); points, Mitsui *et al.* (1986); dashed, Volkova (1980); dot-dashed, Perkins (1993). (*b*) Corresponding calculations of vertical muon flux at sea level (except Mitsui *et al.*) compared to a compilation of data. Kiel data from 1 to 600 GeV (Allkofer *et al.* 1971); diamonds, Durham data from 20 to 400 GeV (Ayre *et al.* 1975); squares, Durham data above 800 GeV (Thompson *et al.* 1977); points, Nottingham data (Rastin 1984); crosses, Houston data (Greene *et al.* 1979); small square with X superimposed, underground data with $E_\mu > 3$ TeV (Ivanenko *et al.* 1985).

importance of kaons as the parents of these neutrinos. Osborne *et al.* (1965) and Cowsik *et al.* (1966) noted that in certain circumstances charged kaons indeed become the dominant source of muon neutrinos at high energy. A consequence of this fact is the importance of the inclusive cross section for kaon production for the flux of ν_μ. One needs the inclusive cross section on nuclear targets for energies of incident nucleons up to several TeV beam energy in the lab.

To a very good approximation, the flux of neutrinos from $\pi^\pm \rightarrow \mu\nu_\mu$ and from $K^\pm \rightarrow \mu\nu_\mu$ is proportional to $Z_{p\pi^\pm}$ and $Z_{pK^\pm}$ respectively, where

$$Z_{p\pi} = \int_0^1 x^\gamma \frac{\mathrm{d}n_{p\pi}}{\mathrm{d}x}\,\mathrm{d}x.$$

Here x is the fractional momentum of the produced pion and $\gamma \approx 1.7$ is the integral spectral index of the primary spectrum. The ratio $R_{K/\pi} = Z_{pK^\pm}/Z_{p\pi^\pm}$ is the appropriate K/pi ratio to characterize neutrino production. Accelerator data for pp collisions give values between 0.10 and 0.13. There is less information on nuclear targets, where the coverage of phase space is not as complete as for proton targets. Values of $R_{K/\pi}$ ranging from 0.10 to 0.15 have been used in the various calculations. For example, Perkins (1993) uses $R_{K/\pi} = 0.10$, Mitsui *et al.* (1986) 0.13 and Volkova 0.15 (Volkova *et al.* 1979). The extension of BGS to high energy (Agrawal *et al.* 1994) also has a relatively high value of $R_{K/\pi}$.

This source of uncertainty is of particular importance because it cannot be easily removed by comparison to high energy muon fluxes. This is because kaons are relatively much more important in the neutrino spectrum than in the muon spectrum. Pions are the dominant source of muons at high energy as well as low, so the calculated muon spectrum is relatively insensitive to $R_{K/\pi}$.

Perkins (1993) has made an interesting comparison between his estimate of the neutrino flux and several others. Figure 3*a* is a similar plot, which shows five calculations of the neutrino flux. Note that plotting $E^2 \times \mathrm{d}N/\mathrm{d}\ln E$ – as Perkins has

Table 3. *Fits to Kamiokande upward muons* (*Frati et al. 1993*)

oscillation	parameters	χ^2 for	10 bins
$\sin^2 2\theta$	δm^2	(1)	(2)
0	0	8.7	18.2
0.5	0.01	14.6	10.3
0.8	0.0046	19.1	10.0
0.5	0.10	19.5	10.9

done – gives a display that reflects the importance of the neutrinos for producing a muon signal. Roughly speaking, one power of E comes from the rising charged current cross section (up to *ca.* 3 TeV) and the other from the increasing muon range (up to *ca.* 500 GeV). Figure 3*b* compares the corresponding vertical muon fluxes with a compilation of the data. Note the relatively large spread in the neutrino fluxes as compared to the muon spectra.

3. Neutrino-induced muons

The large differences among the various neutrino flux calculations in the high energy region limit the certainty with which the neutrino-induced upward muon flux can be calculated. This uncertainty becomes important when one wants to use the measured rate of upward muons to constrain explanations of the contained event puzzle in terms of neutrino oscillations.

For example, the Kamiokande group (Hirata *et al.* 1992) give a preferred 'allowed' set of values for interpretation of their anomalous neutrino flavour ratio for contained events in terms of oscillations in the $\nu_\mu \rightarrow \nu_\tau$ sector: $\delta m^2 = 8 \times 10^{-3}$ eV^2 and $\sin^2 2\theta \approx 0.87$. This value of δm^2 gives the first node in the oscillation probability at $E_\nu \approx 65$ GeV for pathlength $L \approx 10^4$ km:

$$P_{\nu_\mu \rightarrow \nu_\tau} = \sin^2 2\theta \sin^2(1.27 \delta m^2 L(\text{km})/E(\text{GeV})).$$

One would therefore expect a significant reduction of the observed flux of upward, through-going muons relative to the expectation in the absence of oscillations.

Interpretations of the measurements (Mori *et al.* 1991; Becker-Szendy *et al.* 1992*b*), while noting that the conclusions depend on the assumptions, have focused on a particular combination of flux calculation and neutrino cross section; namely, the Volkova (1980) flux and cross sections computed with the EHLQ2 structure functions (Eichten *et al.* 1984). As is apparent from figure 3*a*, the Volkova flux is rather low in the energy range important for neutrino-induced, upward muons. Moreover, the EHLQ2 parametrization of the structure functions underestimates the neutrino cross section by about 12% (Frati *et al.* 1993).

In a recent paper, Frati *et al.* (1993) have systematically investigated the sensitivity of the expected rate of upward muons to the input assumptions. They calculated the expected rate of upward muons in ten equal bins of zenith angle, $-1 < \cos\theta < 0$ and compared the results to the observations of the Kamiokande group (Mori *et al.* 1991). They defined an upward throughgoing muon exactly in accord with the experimental definition (Oyama 1989). Table 3 from Frati *et al.* (1993) summarizes the results for two combinations of flux and cross section and various assumed oscillation parameters for $\nu_\mu \leftrightarrow \nu_\tau$. The two sets of flux and cross section are: (1) Volkova plus EHLQ2, and (2) Bartol (Agrawal *et al.* 1993) plus Owens (1991).

When we use assumption (1) compared in detail to Kamiokande measurements of upward muons, the assumption of no oscillations gives a good fit. In addition, when we use this calculation of the expected rate in 2π sr to determine an excluded region in the neutrino-oscillation space, we find the same excluded region found by IMB (Becker-Szendy *et al.* 1992*b*) from their data with the same input assumptions. This suggests consistency of the IMB and Kamiokande data-sets. With assumption (2), however, we find a better fit with a set of oscillation parameters that leads to significant (*ca.* 15%) reduction in the predicted rate of upward muons.

The same is apparently not true for the Baksan data. Mikheyev (personal communication; Boliev *et al.* 1993) has used the Butkevich *et al.* neutrino flux and a renormalized cross section, essentially equivalent to Owens (1991), and still finds the Kamiokande allowed region of oscillation parameters to be largely excluded.

4. Conclusions

Good progress is being made in understanding differences among the neutrino flux calculations in the low energy region. The Bugaev & Naumov calculation produces an anomalously flat spectrum at low energy as compared to the other calculations. The Lee & Koh calculation has an unphysical value of unity for the $\nu_e/\bar{\nu}_e$ ratio. Sources of these and other differences are under active investigation and should be resolved in the coming months.

Comparison of the muon spectrum from the BGS calculation to new measurements (Circella *et al.* 1993) of *ca.* GeV muons at high altitude (especially 10–20 km, in the peak of the neutrino production region) are quite promising. This and other new measurements of muon spectra at high altitude should make it possible to reduce the uncertainty in the absolute normalization of the neutrino flux calculation in the *ca.* GeV range from ±25% to ±10%.

In a quite plausible scenario (neutrino flux similar to Agrawal *et al.* (1993) or Butkevich *et al.* (1989) and renormalized neutrino cross section) the Kamiokande measurement of upward, throughgoing muons is quite consistent with a $\nu_\mu \rightarrow \nu_\tau$ interpretation of their contained event flavour anomaly. There is indirect evidence (described above) that the IMB data on throughgoing muons is consistent with the Kamiokande data. This is apparently not the case with the Baksan measurement ($-1.0 < \cos\theta < -0.6$) which largely excludes this oscillation scenario, even with the higher flux and cross section. New data from MACRO should be helpful in clarifying the situation.

This talk is based on work done in collaboration with Todor Stanev. I am grateful to M. Honda, H. S. Lee and V. A. Naumov for exchanging information about their calculations. I thank M. Circella for discussions about the MASS measurement of muons at high altitude and David Huber for comparing the BGS muon fluxes with those measurements. I thank Paolo Lipari for helpful discussions and hospitality and the INFN for support at the University of Rome where this talk was prepared. This work is supported by the U.S. Department of Energy.

References

Achar, C. V., *et al.* 1965 *Phys. Lett.* **18**, 196; **19**, 78.

Agrawal, V., Gaisser, T. K., Lipari, P. & Stanev, T. 1994 (In preparation.)

Allkofer, O. C., Carstensen, K. & Dau, W. D. 1971 *Phys. Lett.* B **36**, 425–427.

Ayre, C. A., *et al.* 1975 *J. Phys.* G **1**, 584.

Barr, G., Gaisser, T. K. & Stanev, T. 1989 *Phys. Rev.* D **39**, 3532–3534.

Becker-Szendy, R., *et al.* (IMB Collaboration) 1992*a* *Phys. Rev.* D **46**, 3720–3724.

Becker-Szendy, R., *et al.* 1992*b* *Phys. Rev. Lett.* **69**, 1010–1013.

Bogomolov, E. A. Romanov, V. A., Stepanov, S. V. & Shulakhova, M. S. 1979 LFTI preprint no. 629.

Boliev, M. M., Butkevich, A. V., Chudakov, A. E., Mikheyev, S. P., Skarzhinskaya, N. V. & Zakidyshev, V. N. 1991 In *Proc. 3rd Int. Workshop of Neutrino Telescopes* (ed. M. Baldo Ceolin), pp. 235–245.

Boliev, M. M., Butkevich, A. V., Chudakov, A. E., Mikheyev, S. P., Suvorova, O. V. & Zakidyshev, V. N. 1993 In *Proc. Int. Workshop on* ν_μ/ν_e*-Problem in Atmospheric Neutrinos, Gran Sasso* (ed. V. Berezinsky & G. Fiorentini), pp. 144–155.

Bugaev, E. V. & Naumov, V. A. 1989 *Phys. Lett.* B **232**, 391–397.

Bugaev, E. V., Domogatsky, G. V. & Naumov, V. A. 1986 In *Proc. Japan–U.S. Seminar on Cosmic Ray Muon and Neutrino Physics/Astrophysics using Deep Underground/Underwater Detectors* (ed. Y. Ohashi & V. Z. Peterson), pp. 232–241.

Butkevich, A. V., Dedenko, L. G. & Zheleznykh, I. M. 1988 *Sov. J. Nucl. Phys.* **50**, 90–99.

Circella, M., *et al.* 1993 Mass collaboration. In *Proc. 23rd Int. Cosmic Ray Conf.*, vol. 4, pp. 503–506.

Cowsik, R., Pal, Y. & Tandon, S. N. 1966 *Proc. Indian Acad. Sci.* **63**, 217–243.

Eichten, E., Hinchcliffe, I., Lane, K. & Quigg, C. 1984 *Rev. mod. Phys.* **56**, 579. (Erratum **58** (1986) 1065.)

Eichten, T., *et al.* 1972 *Nucl. Phys.* B **44**, 333–343.

Engel, J., Kolbe, E., Langanke, K. & Vogel, P. 1993 *Phys. Rev.* D **48**, 3048–3054.

Frati, W., Gaisser, T. K., Mann, A. K. & Stanev, T. 1993 *Phys. Rev.* D **48**, 1140–1149.

Gaisser, T. K. & O'Connell, J. S. 1986 *Phys. Rev.* D **34**, 822–825.

Gaisser, T. K., Stanev, T. & Barr, G. 1988 *Phys. Rev.* D **38**, 85–95.

Gaisser, T. K., Stanev, T., Bludman, S. A. & Lee, H. 1983 *Phys. Rev. Lett.* **51**, 223.

Green, P. J., *et al.* 1979 *Phys. Rev.* D **20**, 1598.

Greisen, K. 1960 *A. Rev. Nucl. Sci.* **10**, 63.

Hess, V. F. 1912 (Reprinted in Hillas (1972), pp. 139–147.)

Hillas, A. M. 1972 *Cosmic rays.* (297 pp.) Oxford: Pergamon Press.

Hirata, K. S., *et al.* (Kamiokande Collaboration) 1992 *Phys. Lett.* B **280**, 146–152.

Honda, M. 1993 In *Proc. Calgary Workshop on Atmospheric Neutrinos* (ed. M. C. Goodman), pp. 32–41.

Honda, M., Kasahara, K., Hidaka, K. & Midorikawa, S. 1990 *Phys. Lett.* B **248**, 193–198.

Ivanenko, I. P., *et al.* 1985 In *Proc. 19th Int. Cosmic Ray Conf.* (*La Jolla*) **8**, 210.

Lee, H. & Bludman, S. A. 1988 *Phys. Rev.* D **37**, 122–125.

Lee, H. & Koh, Y. S. 1990 *Nuovo Cimento* B **105**, 883–888.

Markov, M. A. 1960 In *Proc. 1960 Annual Int. Conf. on High Energy Physics at Rochester* (ed. E. C. G. Sudarshan, J. H. Tinlot & A. C. Melissinos).

Markov, M. A. & Zheleznykh, I. M. 1961 *Nucl. Phys.* **27**, 385–394.

Menon, M. G. K., *et al.* 1967 *Proc. R. Soc. Lond.* A **301**, 137–157.

Merenyi, R., *et al.* 1992 *Phys. Rev.* D **45**, 743–751.

Minorikawa, Y. & Mitsui, K. 1984 *Nuovo Cimento* **41**, 333–338.

Mitsui, K., Minorikawa, Y. & Komori, H. 1986 *Nuovo Cimento* C **9**, 995–1020.

Mori, M., *et al.* (Kamiokande Collaboration) 1991 *Phys. Lett.* B **270**, 89.

Osborne, J. L., Said, S. S. & Wolfendale, A. E. 1965 *Proc. phys. Soc.* **86**, 93–99.

Owens, J. F. 1991 *Phys. Lett.* B **266**, 126.

Oyama, Y. 1989 Experimental study of upward-going muons in Kamiokande. (207 pp.) University of Tokyo Thesis (ICR-Report-193-89-10).

Perkins, D. H. 1993 *Nucl. Physics* B **399**, 3–14.

Rastin, B. C. 1984 *J. Phys.* G **10**, 1609.

Reines, F., *et al.* 1965 *Phys. Rev. Lett.* **15**, 429.

Reines, F. 1967 *Proc. R. Soc. Lond.* A **301**, 125–135.

Ronga, F. 1993 In *Proc. 5th Int. Workshop on Neutrino Telescopes. Venice.*

Staner, T. 1993 *In Proc. Int. Workshop on Muon Neutrino/Electron Neutrino Problems in Atmospheric Neutrinos* (ed. V. Berezinsky & G. Fiorentini), pp. 4–24.

Tam, A. C. & Young, E. C. M. 1970 In *Proc. 11th. Int. Cosmic Ray Conf.* (*Budapest, 1969*). *Acta Physica Academiae Scientiarum Hungaricae* **29** (4), 307–312.

Thompson, M. G., Thornley, R. & Whalley, M. R. 1977 *J. Phys.* G **3**, L39.

Volkova, L. V. 1980 *Sov. J. Nucl. Phys.* **31**, 784–789.

Volkova, L. V. 1989 In *Cosmic gamma rays, neutrinos and related astrophysics* (ed. M. M. Shapiro & J. P. Wefel). NATO ASI vol. 270, p. 139.

Volkova, L. V., Zetsepin, G. T. & Kuz'michev, L. A. 1979 *Sov. J. Nucl. Phys.* **29**, 645–651.

Zatsepin, G. T. & Kuzmin, V. A. 1962 *Sov. J. Nucl. Phys.* **14**, 1294.